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Fitting Multiphasic Logistic Functions to the Lactation Curves of Gir x Friesian Crossbred Dairy Cattle in Malaysia

(Pemadanan Fungsi Berbilang Fasa kepada Lengkung Laktasi Kacukan Lembu Tenusu Gir x Friesian di Malaysia)

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ABSTRACT

The data used in this study consisted of milk yield (kg) taken at approximately fortnightly intervals from Gir x Friesian crossbred dairy cattle raised at Institut Haiwan Kluang, Malaysia. The data were first edited, smoothed and then fitted with mono-, di- and triphasic logistic functions. In general, parameter estimates for the first lactation were reasonable. However, for the second lactation the estimates were erratic and unreasonable because this was the atypical lactation for which the multiphasic functions were obviously unsuitable. Residual mean squares for the di- and triphasic functions of the first lactation were very similar (0.0002 and 0.0004, respectively) and smaller for the monophasic function (0.0894). For the second lactation, residual mean squares for the triphasic function (0.001) was the lowest compared to those for the mono- and diphasic functions (0.0345 and 0.0315). For the first lactation, the monophasic function did not fit the data well because it had large residuals. The di- and triphasic functions were almost similar in fitting the lactation and had low residuals. For the second lactation, both the mono- and diphasic functions did not fit the data very well and had rather large residuals. The triphasic function was the most fitting and had small residuals. Derived functions were generally lower for the first lactation than the second lactation: initial milk yields (4.88 to 6.0 kg versus 9.9 to 11.8 kg); peak milk yields (5.8 to 9.6 kg versus 12.8 to 15.7 kg) and 305-day milk yields (1147.7 to 1328.6 kg versus 1687.4 to 2296.1 kg).

Keywords: Gir x Friesian crossbred cattle; lactation; milk yield; multiphasic logistic functions

ABSTRAK

Data hasil susu (kg) yang diguna dalam kajian ini telah ditimbang lebih kurang setiap dua minggu daripada lembu kacukan tenusu Gir x Friesian yang diternak di Institut Haiwan Kluang, Malaysia. Data ini terlebih dahulu disunting dan dilicinkan sebelum dipadankan dengan fungsi logistik mono-, dwi- dan trifasa. Secara am, aggaran parameter untuk laktasi pertama adalah munasabah. Tetapi, anggaran untuk laktasi kedua adalah tidak menentu dan tidak munasabah kerana laktasi ini luar biasa dan fungsi logistik tidak sesuai dipadankan kepadanya. Min kuasa dua ralat untuk fungsi dwi- dan trifasa bagi laktasi pertama adalah hampir sama (0.0002 dan 0.0004) dan lebih kecil untuk fungsi monofasa (0.0894). Bagi laktasi kedua, min kuasa dua ralat untuk fungsi trifasa (0.001) adalah paling rendah jika dibandingkan dengan fungsi mono- dan dwifasa (0.0345 dan 0.0315). Bagi laktasi pertama, fungsi monofasa tidak padan dengan data kerana ia mempunyai ralat yang besar. Fungsi dwi- dan trifasa adalah hampir sama padan untuk laktasi ini dan mempunyai ralat yang rendah. Bagi laktasi kedua, fungsi mono- dan dwifasa tidak padan pada data dengan baik dan mempunyai ralat yang besar. Fungsi trifasa adalah yang paling padan dan mempunyai ralat yang rendah. Secara am, fungsi-fungsi terbitan adalah lebih rendah bagi laktasi pertama daripada laktasi kedua: hasil susu awal (4.88 hingga 6.0 kg berbanding 9.9 hingga 11.8 kg); hasil susu kemuncak (5.8 hingga 9.6 kg berbanding 12.8 hingga 15.7 kg) dan hasil susu 305 hari (1147.7 hingga 1328.6 kg berbanding 1687.4 hingga 2296.1 kg).

Kata kunci: Fungsi pelbagai fasa logistik; hasil susu; kacukan lembu Gir x Friesian; laktasi

Introduction

In Malaysia, crossbreeding of dairy cattle between *Bos taurus* and *B. indicus* breeds started as early as the 1930s (Sivarajasingam 1975). At that time, however, there was no organized breeding programme. It was only in 1963 that crossbreeding between the two sub-species became organized (Wan Hassan 1990). Another milestone in dairy production in Malaysia occurred in 1974 when the Department of Veterinary Services started importing foreign breeds from New Zealand and Australia. One of the crossbreds formed in Malaysia was between the Gir (*B. indicus*) and the Friesian (*B. taurus*) breeds.

Over the years, various mathematical functions have been fitted to lactations. The most common is the incomplete gamma function used by Ferris et al. (1985), Nur Farydah (2002), Rao and Sundaresan (1979), Varona et al. (1998) and Wood (1980, 1976, 1969, 1968, 1967). Polynomial regression equations have also been used for dairy cattle (McCraw & Butcher 1976) and dairy goats (Majid 1985).

The multiphasic logistic function is an example of an empirical or functional model which is characterized by having less parameters and easier to handle mathematically than models that are mechanistic (Steri 2009). This function was first developed by Koops (1986) to study the growth of animals and man. Differentiating this function with respect to time yielded the multiphasic logistic functions presently used to model lactation curves. Its application to dairy cattle lactations was first introduced by Grossman and Koops (1988). Gipson and Grossman (1989) then applied it to dairy goat lactation.

In Malaysia, the fitting of lactation curves with multiphasic logistic functions was first performed by Farah (2004) and Faridah (2004) but they were only successful with the monophasic function. This was followed by Hairun Nisa (2007) and Suhaili (2007) who were partially successful in fitting up to the triphasic function. The present study hopes to improve on the work of the previous researchers.

Materials and Methods

Lactation Data

The data used in this study were collected from crossbred Gir x Friesian dairy cattle raised at Institut Haiwan, Kluang, Malaysia. The data were stored in the record-keeping software system called DairyCHAMP 1.1 (Dairy Computerized Health and Management Programme). Among the information contained in the system were breed of cow, identification number, date of birth of dam, paternal breed, maternal breed, lactation number, date of test, test milk weight, maximum milk yield, expected milk yield, expected 305-day milk yield, dry-off date and lactation length. The data consisted of milk yield (kg) taken at approximately fortnightly intervals.

The available data were first edited before being subjected to statistical analysis. Some records were omitted from the data set for the following reasons: unknown genotype, unknown parental breed, lactations with less than six milk samples, lactation number greater than six and lactations with records starting more than 35 days in milk.

Statistical Analyses

Lactations were smoothed using PROC LOESS of the SAS package (SAS 1985). The moving average algorithm of this procedure created a smooth curve in place of the fluctuating mean milk yields of each lactation. Mean milk yields and those obtained by smoothing using PROC LOESS at 20, 40, …, 280, 300 days are shown in Table 1. The smoothed lactations were then fitted with the multiphasic logistic functions of the form $y_t = \Sigma \{a_i b_i [1 – \tanh^2 (b_i (t - c_i))]\}$, where y_t is milk yield at time t, $a_i b_i$ is peak milk yield, tanh is the hyperbolic tangent, b_i is the lactation parameter at the i^{th} phase, t is days in milk and c_i is time of peak milk yield. The parameters of the equations were estimated using PROC NLIN (non-linear procedure) of the SAS package (SAS 1985). The Gauss-Newton method was used in parameter estimation and the number of iterations was limited to 100.

Derived functions obtained using the estimates were initial yield, peak yield and 305-day yield. Initial yield was estimated as $y_t = \Sigma \{a_i b_i [1 – \tanh^2 (b_i (t - c_i))]\}$ with t=0, peak yield as $a_i b_i$ and 305-day yield as $MY_{305} = \Sigma \{a_i [\tanh(b_i(305 - c_i)) – \tanh(b_i(0 - c_i))]\}$. Residual values, which is the difference between the predicted and mean smoothed values, were used as a measure of goodness-of-fit of the multiphasic models.

TABLE 1. Mean milk weights and smoothed values obtained by using PROC LOESS of SAS for Gir x Friesian lactations[1]

Days in milk	1st Lactation		2nd Lactation	
	Mean (kg)	Smoothed (kg)	Mean (kg)	Smoothed (kg)
20	5.10	5.31	10.10	9.59
40	6.10	5.84	8.20	8.95
60	6.40	6.13	8.30	8.46
80	6.20	6.21	8.20	8.10
100	6.00	6.03	8.00	7.93
120	5.70	5.68	7.80	7.73
140	5.30	5.34	7.40	7.47
160	5.00	5.02	7.00	7.21
180	4.90	4.79	7.10	6.85
200	4.40	4.69	6.60	6.39
220	4.80	4.64	5.70	5.90
240	4.80	4.57	5.30	5.56
260	4.40	4.45	5.30	5.32
280	4.40	4.29	5.40	5.19
300	4.00	4.08	5.10	5.18

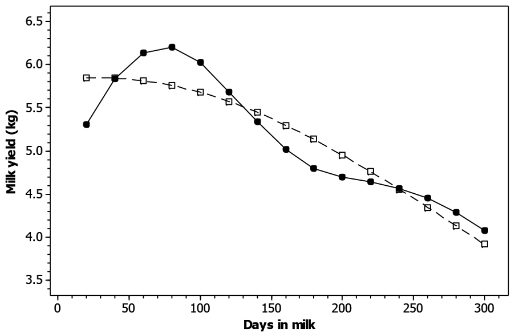
[1]Number of lactations involved were 17 and 12 for the first and second lactations, respectively

RESULTS AND DISCUSSION

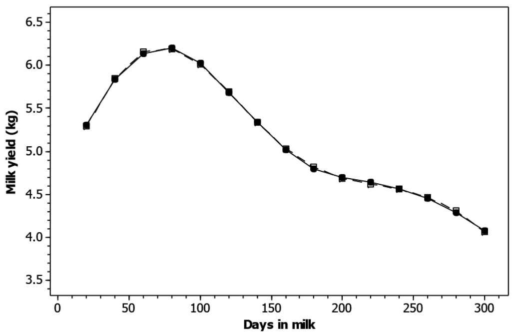
Figure 1(a), 1(b) and 1(c) shows that the first lactation had a standard lactation curve characterized by an initial low value, increasing towards peak milk yield and finally declining gradually towards the end of lactation. However, as indicated in Figure 3(a), 3(b) and 3(c), the second lactation was an atypical lactation that had no inclining phase, no peak and had only declining phase. Atypical lactation curves have been observed in cattle (Congelton & Everett 1981; Shanks et al. 1980), sheep (Cappio-Borlino et al. 1997) and goats (Macciotta et al. 2008). The absence of a peak in such lactations can be ascribed to either the peak occurring before parturition or too soon after parturition such that the first milk yield was recorded after the peak. Mean squares from analyses of variance for the first and second lactations of Gir x Friesian cattle are shown in Table 2. The effect of the fitted model in the mono-, di- and triphasic functions were all significant (p<0.01). In both lactations, the triphasic function had the lowest mean squares for the residual effect, indicating that it is the most suitable for fitting both lactations.

The parameter estimates for the first and second lactations are shown in Table 3. The estimates for the mono-, di- and triphasic functions of the first lactation were reasonable and all had positive values. However, the estimates for the second lactation were erratic and unreasonable. This must be due to the fact that it is atypical lactation without an increasing phase and a peak. Some of the estimates had negative values, indicating that the multiphasic function may not be suitable for fitting atypical lactations. Similar changes in the sign of the estimates for atypical lactations have also been described by Macciotta et al. (2008).

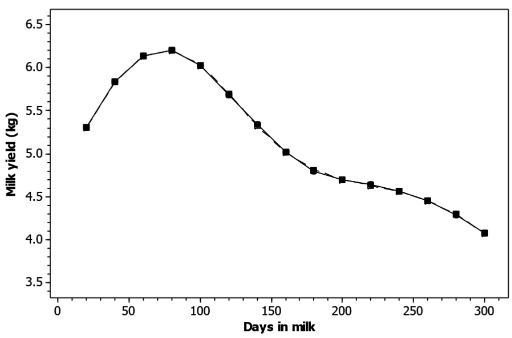
Figure 1(a), 1(b) and 1(c) shows the smoothed curve of the first lactation fitted with mono-, di- and triphasic functions, respectively. The monophasic function had no peak and did not fit the curve well at several phases of the lactation (Figure 1(a)). Residual values in Figure 2 shows that it tended to underpredict milk yield from 20 through 40 days, overpredict from 40 through 130 days, underpredict again from 130 through 240 days and finally overpredict from 240 through 300 days. The diphasic and triphasic functions (Figure 1(b) and 1(c)) both fitted the curve well. The residual values for both functions were low and very similar to each other, implying that at least for this particular lactation, the diphasic function was just as good as the triphasic function.



(a)



(b)



(c)

FIGURE 1. First lactation of Gir x Friesian cattle fitted with (a) monophasic, (b) diphasic and (c) triphasic logistic function (● smoothed, □ predicted)

TABLE 2. Mean squares from analyses of variance for monophasic, diphasic and triphasic functions for the first and second lactations of Gir x Friesian cattle

Source of Variation	d.f.[1]	1st Lactation			2nd Lactation		
		Monophasic	Diphasic	Triphasic	Monophasic	Diphasic	Triphasic
Model	a	134.0000**	67.1706**	44.7807**	258.3000**	129.2000**	86.1582**
Residual	b	0.0894	0.0004	0.0002	0.0345	0.0315	0.0010

[1]Degrees of freedom for Model and Residual were, respectively, 3 and 12 for monophasic, 6 and 9 for diphasic and 9 and 6 for triphasic
** p<0.01

TABLE 3. Parameter estimates (± standard errors) of the monophasic, diphasic and triphasic functions for the first and second lactations of Gir x Friesian cattle

Parameters	1st Lactation			2nd Lactation		
	Monophasic	Diphasic	Triphasic	Monophasic	Diphasic	Triphasic
a1	2418.70 ± 547.40	620.50 ± 29.81	232.30 ± 90.72	9310.40 ± 6034.90	29.96 ± 393.30	-791.40 ± 211.40
b1	0.0024 ± 0.0005	0.0086 ± 0.0002	0.012 ± 0.001	0.0016 ± 0.0003	0.20 ± 0.92	0.009 ± 0.001
c1	29.00 ± 52.59	58.56 ± 1.27	84.01 ± 8.27	-420.70 ± 317.10	28.71 ± 6.84	51.30 ± 9.54
a2		585.30 ± 39.46	811.00 ± 80.32		4415.90 ± 1248.30	3185.80 ± 473.1
b2		0.0066 ± 0.0004	0.0053 ± 0.0005		0.0022 ± 0.0004	0.0051 ± 0.0007
c2		272.40 ± 2.33	250.00 ± 7.77		-100.70 ± 99.88	20.07 ± 20.37
a3			194.70 ± 61.93			304.30 ± 199.00
b3			0.013 ± 0.002			0.012 ± 0.003
c3			13.27 ± 10.76			363.10 ± 31.94

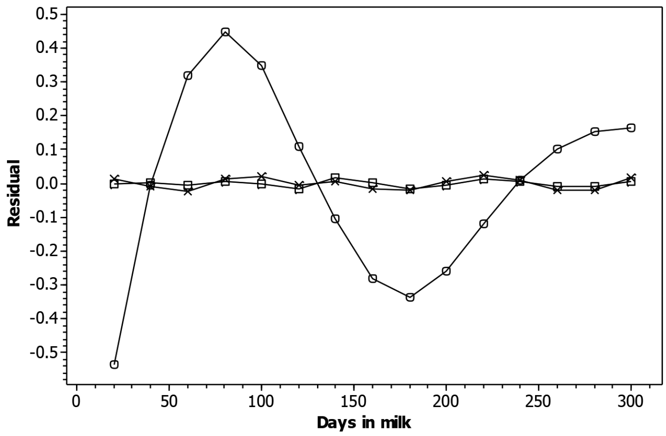


FIGURE 2. Residual values for the first lactation of Gir x Friesian cattle fitted with monophasic (○), diphasic (×) and triphasic (□) logistic functions

Figure 3(a), 3(b) and 3(c) shows the smoothed curve of the second lactation fitted with mono-, di- and triphasic functions, respectively. The triphasic function fitted the lactation best and had the lowest residual values which were fairly randomly distributed (Figure 4). The mono- and diphasic functions did not fit the lactation as well as the triphasic function and had larger residuals. The monophasic function tended to overpredict milk yield in the initial phase of the lactation, underpredict from 40 through 110 days, overpredict from 110 through 200 days, underpredict from 200 through 280 days and finally overpredict from 280 days to the end of the lactation. The diphasic function predicted milk yield quite closely up to 60 days, underpredicted from 60 through 100 days and then followed closely the pattern showed by the monophasic function.

Derived functions calculated from the parameter estimates were initial yield, peak milk yield and 305-day yield (Table 4). Initial yields for the first lactation were estimated from 4.88 to almost 6 kg and were lower than between 9.9 and 11.8 kg estimated for the second lactation. Peak milk yields for the first lactation were estimated at between 5.8 and 9.6 kg; estimates for the second lactation were higher and ranged from 12.8 to 15.7 kg. It must be cautioned that the second lactation had no peak so the estimated peak must be a theoretical value that occurred before the start of lactation. 305-day milk yield estimates for the first lactation were from 1147.7 to 1328.6 kg and for the second lactation from 1687.4 to 2296.1 kg.

Conclusion

In the present study, the first lactation represents a standard lactation with an ascending phase, a peak and a decreasing phase while the second lactation represents the atypical lactation with no ascending phase, no peak and only a descending phase. Due to the nature of the multiphasic logistic functions, the estimates of parameters were more logical for the first lactation. The estimates for the second lactation, however, were erratic and unreasonable. As a result, the functions tended to fit the first better than the second lactation. For the first lactation, based on the fitted curve and the residuals, the diphasic function was almost as good as the triphasic function. However, for the second lactation, it was necessary to fit the triphasic function as the diphasic function had large residuals.

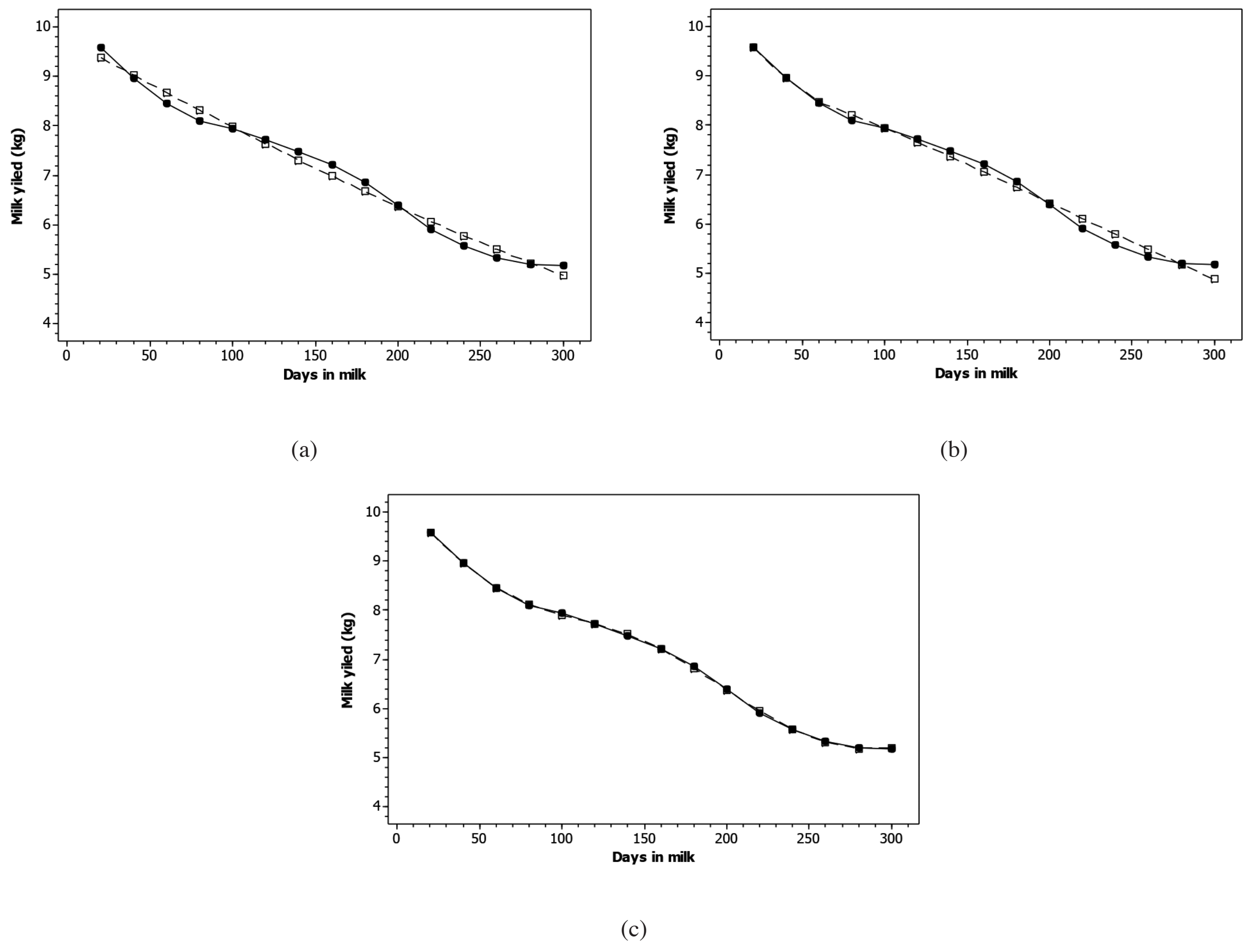


FIGURE 3. Second lactation of Gir x Friesian cattle fitted with (a) monophasic, (b) diphasic and (c) triphasic logistic function (• smoothed, □ predicted)

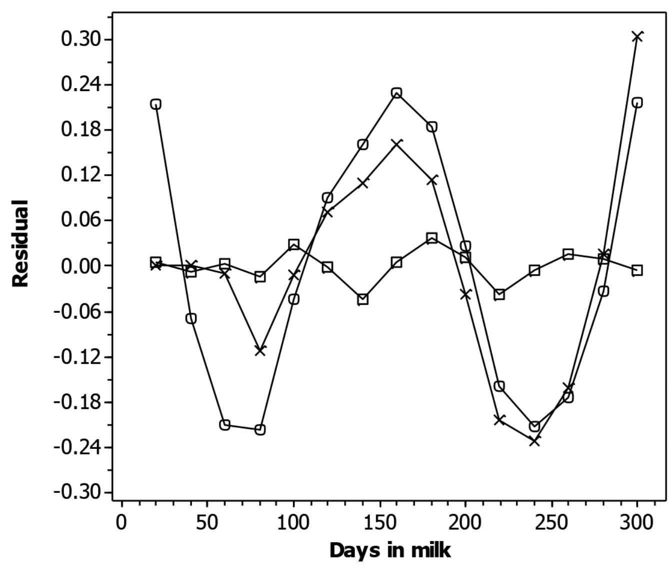


FIGURE 4. Residual values for the second lactation of Gir x Friesian cattle fitted with monophasic (○), diphasic (×) and triphasic (□) logistic functions

TABLE 4. Functions derived from parameter estimates for the first and second lactations of Gir x Friesian cattle

Phase		Derived functions					
		Initial yield[1] (kg)		Peak yield[2] (kg)		305-day yield[3] (kg)	
		1st Lact.	2nd Lact.	1st Lact.	2nd Lact.	1st Lact.	2nd Lact.
Monophasic	1	4.88	9.89	5.80	14.89	1233.54	1689.83
Diphasic	1	3.98	2.53	5.34	5.99	577.06	44.04
	2	1.97	9.24	3.86	9.71	570.67	2252.10
	Total	5.95	11.77	9.20	15.70	1147.73	2296.14
Triphasic	1	1.42	-5.93	2.79	-7.12	353.09	-925.94
	2	2.19	16.08	4.30	16.25	793.97	2546.64
	3	2.09	1.54	2.53	3.65	181.55	48.69
	Total	5.70	11.71	9.62	12.78	1328.61	1687.39

[1]Estimated from $y_t = \sum \{a_i b_i [1 - \tanh^2 (b_i (t - c_i))]\}$ with t=0.
[2]Estimated from $a_i b_i$.
[3]Estimated from $MY_{305} = \sum \{a_i [\tanh(b_i(305 - c_i)) - \tanh(b_i(0 - c_i))]\}$

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